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Partition of a Unity on Infinite-Dimensional Manifold of the Lipschiz Class Lip^k

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Abstract—In the present paper we prove a criterion of Lip^k-paracompactness for infinite-dimensional manifold M modeled in nonnormable topological vector Fréchet space F. We establish that a manifold is Lip^k-paracompact if and only if the model space F is paracompact and Lip^k-normal. We prove a sufficient condition for existence of Lip^k-partition of a unity on a manifold of class Lip^k.

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INTRODUCTION

A topological Hausdorff space M with countable base is called manifold if any its point x has a neighborhood $N(x)$ homeomorphic to an open set (usually, ball B) in model space E, which has certain additional structure (for instance, it is topological linear space). The pair consisting of the neighborhood $N(x)$ and homeomorphism $\phi_x : N(x) \mapsto B \subset E$ is called a map, and a collection of maps covering M is an atlas. If (N_1, ϕ_1) and (N_2, ϕ_2) are two maps with non-empty intersection $N = N_1 \cap N_2$, then there is defined gluing mapping $\phi_2 \circ \phi_1^{-1}$ satisfying additional restrictions. Thus, if $E = R^n$ and gluing mappings are differentiable, then the manifold is called differentiable. There are well-known smooth and analytic manifolds. If the model space is finite-dimensional, then the corresponding manifolds are called finite-dimensional. Theory of that manifolds is extremely extensive. Here we restrict our references by the fundamental monograph [1]. In the present work we consider manifolds with infinite-dimensional model spaces.

Two types of infinite-dimensional linear spaces are most known: Banach spaces and Fréchet spaces. We study here the case, where E is nonnormable Fréchet space. Analogous problems for manifolds with Banach model spaces are investigated in [2].

According to monographs [3] and [4], we apply for description of the class of manifolds under consideration parametric curves in nonnormable Fréchet space E, i.e., mappings $c : R \mapsto E$ and their derivatives. In what follows we write "a curve" instead of "parametric curve". Note that space of smooth curves $C^\infty(R, E)$ depends only on main system of bounded sets in E. Topology of this space is Mackey topology. A. Frolicher and A. Kriegl [4] introduced so-called convenient vector spaces. They are conjugated to Fréchet spaces, and their topology is topology of the Mackey closure. It is defined as final topology with regard to all convergent Mackey sequences $S : N_\infty = N \cup \{\infty\} \to E$ [4]. A subset U of convenient space E is open in this topology if and only if the following requirement is fulfilled. If $x_\infty \in U$ is limit of a convergent Mackey sequence in E, then there exists a number $n \in N$ such that $x_k \in U$ for $k \geqslant n$ [4]. Open and closed in this topology sets are called μ-open and μ-closed correspondingly.

Furthermore, let E and F be two convenient vector spaces, and E' is conjugated to E space. A curve $\alpha : R \to E$ is called differentiable if derivative $\alpha'(t) = \lim\limits_{s \to 0} \frac{\alpha(t+s) - \alpha(t)}{s}$ exists for any t. It is called

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